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THESIS

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In a University Faculty

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ABSTRACT

A Markov Model is developed to study the distribution of faculty in a university. The purpose of this paper is to determine the underlying factors that control the distribution of faculty members from one time period to the next. Maintainable and realizeable distributions are defined. The effects of appointment policies and promotion and retirement policies in controlling distributions are discussed. The model is tested by comparing the predicted output with real data.

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I. INTRODUCTION

In any graded manpower system, one of the principal concerns of top level management is the control of the distribution of personnel among the various ranks to enhance the operation of the organization. The means available to management for controlling this hierarchy are as follows:

(a) The number of personnel recruited into each grade at each point in time.

(b) The number of persons promoted (or demoted) from one grade to the next within the organization.

(c) The number of personnel dismissed from the organization or induced to leave the system.

In most organizations, distribution control by means of demotions and dismissals are last resort measures that are employed only under extreme circumstances. Control by (a) alone has many attractions since it is least disruptive to those persons already in the system. However, control of unwanted growth at the top is usually achieved by means of reduced promotion rates and induced retirement.

The purpose of this paper is twofold: first to design a mathematical model that quantitatively describes the movement of university faculty between ranks and to study the effects that promotion, appointment and retirement policies have on future rank structures of faculty; and second to test this model in a case study by comparing predicted results with actual data. It has been recognized for some time that promotion

practices and retirement policies have a profound effect on future faculty distributions. Few people would agree on an ideal distribution of faculty within a college or department, although most persons would agree that certain distributions are undesireable. For instance, a distribution of faculty with all members in full professor rank could be argued as undesirable both from an economic standpoint and because such a distribution of faculty would greatly reduce the infusion of new people and ideas into the institution. No attempt will be made in this paper to prescribe a desired distribution of faculty. Rather, this paper will be concerned with a quantitative analysis of the types of faculty distributions that can be realized, given certain promotion and retirement policies, and how these distributions will be effected by alterations to these policies.

The following assumptions are made throughout this paper:

(a) The organization being studied (college, department of a college, or university) has a faculty whose total number is constant from year to year.

(b) The movement of faculty between ranks from one year to the next is a Markov type process in that it is possible to deduce the future distributions from knowledge of the present rank distribution. Although this assumption may seem severe and in some circumstances unrealistic, the results of the prediction model show that it is a reasonable one to make when trying to predict over periods of 5 years.

This paper begins with a general description of the model, followed by a detailed explanation of its uses in studying the effects of policy

decisions. This is followed by a case study of the College of Engineering at the University of California, Berkeley, where actual data is used in estimating the Promotion Matrix.

II. THE MARKOV MODEL

Consider a discrete time Markov model consisting of m states where state j is the rank of a faculty member of a university, $j = 1,2...,m$. These m states can be divided into two categories namely: n transient states and m-n absorbing states. The sixteen states in the case study are defined as follows:

States	Description
1 - 4	Assistant Professor steps 1 - 4
5 - 7	Associate Professor steps 1 - 3
8 - 12	Full Professor steps 1 - 5
13	Full Professor, overscale
14	Retired Professor
15	Resigned Professor
16	Deceased Professor ,

where states 1 - 13 are the $n = 13$ transient states and states 14 - 16 are the $m-n = 3$ absorbing states.

Let P_{ij} be the fraction of faculty in state j on 1 July of a particular year that were in state i as of 1 July of the previous year. The date 1 July is taken for administrative bookkeeping reasons. Since demotions are extremely rare in a university system, we take $P_{ij} = 0$ for $j < i$. The state corresponding to retired, resigned and deceased faculty are considered to be absorbing states; thus, $P_{ii} = 1$ for i = 14, 15, and 16. Another constraint on the P_{ij}'s, is that $0 < P_{ii} < 1$ for $i = 1,...,13$ which simply requires that a positive fraction of the faculty in any given state will remain in that state for the succeeding time period, and that no

active faculty member can remain in a given state indefinitely. Because the state space describes all possible states of nature for faculty, this implies that:

$$\sum_{j=1}^{m} P_{ij} = 1 , \quad i = 1,2,\ldots m .$$

The fractions, P_{ij}, are conveniently set out in a transition matrix, Q, as follows:

$$Q = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \ldots & P_{1,n+1} & P_{1,n+2} & P_{1,m} \\ 0 & P_{22} & P_{23} & \ldots & P_{2,n+1} & P_{2,n+2} & P_{2,m} \\ 0 & 0 & P_{33} & \ldots & P_{3,n+1} & P_{3,n+2} & P_{3,m} \\ . & . & . & \ldots & . & . & . \\ . & . & . & \ldots & . & . & . \\ . & . & . & \ldots & 1 & 0 & 0 \\ . & . & . & \ldots & 0 & 1 & 0 \\ . & . & . & \ldots & 0 & 0 & 1 \end{bmatrix} .$$

The rows of the Q matrix correspond to the present state and the columns correspond to the state of the system one year from now.

Consider a new matrix P formed from Q by deleting the last m-n rows and columns. This new matrix, called the Promotion Matrix, specifies the transition processes of active faculty members. P is a square (n x n) matrix with:

$$\sum_{j=1}^{n} P_{ij} \leq 1 , \quad i = 1,2,\ldots,n ,$$

and at least one row of the P matrix sums to less than one.

Let $X^*(t)$ be an n-dimensional row vector, $X^*(t) = (x_1^*(t) \ldots x_n^*(t))$, where $x_i^*(t)$ represents the number of faculty in state i at time t. Then $X^*(t)$ is a vector representation of the distribution of faculty. Further, let the appointments in year t be represented by an n-dimensional row vector $(F^*(t) = (f_1^*(t) \ \ldots f_n^*(t))$, where $f_i^*(t)$ is the number of faculty appointed to position i in year t and we assume that there is no constraint on the availability of persons to fill open positions.

The distribution of faculty at time t + 1 is determined from the faculty distribution at time t, the promotion matrix, and the appointment policy at time t + 1. In vector matrix notation this becomes:

$$X^*(t+1) = X^*(t) \cdot P + F^*(t+1). \qquad (1)$$

Since it is not possible to have a negative number of faculty in any rank or to make negative appointments to a rank (dismissing of faculty is considered as a resignation in the transition matrix), it is necessary that $x_i^*(t)$ and $f_i^*(t) \geq 0$ for $i = 1,2,\ldots,n$.

If the total faculty size is to remain constant from one year to the next, the total number of personnel leaving the system each year must be replaced by new faculty. Since the number leaving the system in any given year is a function of the faculty distribution during the year, the total number of positions available for appointment in year t + 1 is given by:

$$\sum_{i=1}^{n} f_i^*(t+1) = X^*(t) \cdot (I-P) \cdot \underset{\sim}{1} \ , \qquad (2)$$

where $\underset{\sim}{1}$ is an n-dimensional column vector of 1's, and I is an (n x n) identity matrix.

III. EFFECT OF APPOINTMENT POLICIES

Since appointment policies are the most desired means of controlling faculty distributions, we investigate the extent to which distributions can be controlled by this means. Recall that the total number of faculty remains constant from one time period to the next. Under this condition we can see from equation 2 that the number of positions available for new appointments in a given year is a function of the previous year's faculty distribution. In order to better understand the extent to which distributions are restricted by the transition process, three subsets of the set of possible distributions are defined and discussed, namely:

A) The Attainable set - those distributions of faculty which can be attained from at least one previous distribution.

B) The Maintainable set - those faculty distributions which can be maintained from one time period to the next.

C) The Containment set - those distributions of faculty which can be attained infinitely often, but not necessarily in consecutive periods. This is followed by a discussion of appointment policies and how they can be used to achieve desired distributions.

In studying the distribution of faculty by rank, it is more convenient to consider the fraction of the total number of faculty in a particular rank rather than the absolute number of faculty in that rank. For this reason we define a new vector $X(t) = (x_1(t), \ldots x_n(t))$, where

$$x_i(t) = \frac{x_i^*(t)}{\sum_j x_j^*(t)} .$$

Note that in the remainder of this paper, if no limits are given on the summation sign it is assumed that the summation is over all transient states and the dummy variable of summation is omitted when no ambiguity exists. Define the set,

$$Y = \{ X(t) \mid X(t) \cdot \underset{\sim}{1} = 1, \; x_i(t) \geq 0 \text{ for all } i \} .$$

The set Y then contains all faculty distributions.

A. THE ATTAINABLE SET

The set of all distributions is the fundamental simplex in n-space (see Figure 1). The question naturally arises, do there exist distributions to which we can return in some future year but which might not be maintained from year to year? If the time period for repetition is sufficiently small and the intervening distributions sufficiently close to the desired distribution, the set of such distributions would be of great practical interest.

A necessary condition for repeating a distribution is that the distribution must be reachable from at least one distribution in Y. The set of all such reachable distributions we call the Attainable set A. Since it is only possible to make non-negative appointments, $F^*(t) \geq 0$ for all t, and following equation 1 we define A as follows:

$$A = \{ X \mid X \geq Z \cdot P \; ; \; X, Z \in Y \} \qquad .$$

Although this definition characterizes the Attainable set, it does not explicitly describe the boundaries of this set. We now seek a characterization of A which describes its boundaries.

As mentioned above, the total number of new appointments available in any year is a function of the distribution of faculty the previous year. Define a column vector, $W = (W_1, \ldots, W_n)'$, as follows:

$$W_i = \sum_{k=n+1}^{m} P_{ik} \quad , \quad i = 1,2,\ldots,n.$$

Then W_i is the proportion of faculty in rank i which annually leave the institution.

Consider the extreme points of the set of all distributions, Y, represented by the vectors $\{e_i\}$, the n-dimensional unit vectors, Since the extreme points are contained in Y, any distributions reachable from the extreme points must be elements of A. Define a vector $F(t) = (f_1(t), \ldots, f_n(t))$ where $f_i(t)$ is the <u>fraction</u> of new appointments to rank i at time t. Then equation 1 can be written as follows:

$$X(t+1) = X(t) \cdot P + X(t) \cdot W \cdot F(t+1) \quad . \tag{3}$$

Let $X(0) = e_i$. Then the distributions that are attainable at time $t = 1$ are given by:

$$X(1) = e_i \cdot P + e_i \cdot W \cdot F(1) \quad , \tag{4}$$

$$= P^{(i)} + W_i \cdot F(1) \quad , \tag{5}$$

where $P^{(i)}$ is the ith row of P and F(1) is any appointment vector.

Consider any other starting point $Z = (Z_1, \ldots, Z_n)$. The coordinates of Z can be written $\Sigma Z_i \cdot e_i$. Substituting this representation of Z into equation 4 for e_i we get:

$$X(1) = \Sigma Z_i \cdot e_i \cdot P + \Sigma Z_i \cdot e_i \cdot W \cdot F(1) \quad ,$$

$$= \Sigma Z_i \cdot P^{(i)} + \Sigma Z_i \cdot W_i \cdot F(1) ,$$

$$= \Sigma Z_i \{ P^{(i)} + W_i \cdot F(1) \} . \tag{6}$$

Since the Z_i's sum to one, any attainable distribution given by equation 6 is a convex combination of the distributions described by equation 5. Thus the Attainable set, A, is the convex hull of the points with co-ordinates:

$$P^{(i)} + W_i \cdot e_j \quad (j=1,2,\ldots,n;\ i=1,2,\ldots,n) \tag{7}$$

At this point we consider an example to illustrate the Attainable set. In order to illustrate the problem diagramatically, we consider the following reduced state space:

State 1 = Assistant Professors

2 = Associate Professors

3 = Full Professors

4 = Retired, resigned or deceased Professors.

A typical transition matrix may be as follows:

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} .5 & .4 & 0 & .1 \\ 0 & .6 & .3 & .1 \\ 0 & 0 & .7 & .3 \\ 0 & 0 & 0 & 1. \end{bmatrix} \end{array}$$

The promotion matrix would then be:

$$P = \begin{bmatrix} .5 & .4 & .0 \\ .0 & .6 & .3 \\ .0 & .0 & .7 \end{bmatrix}$$

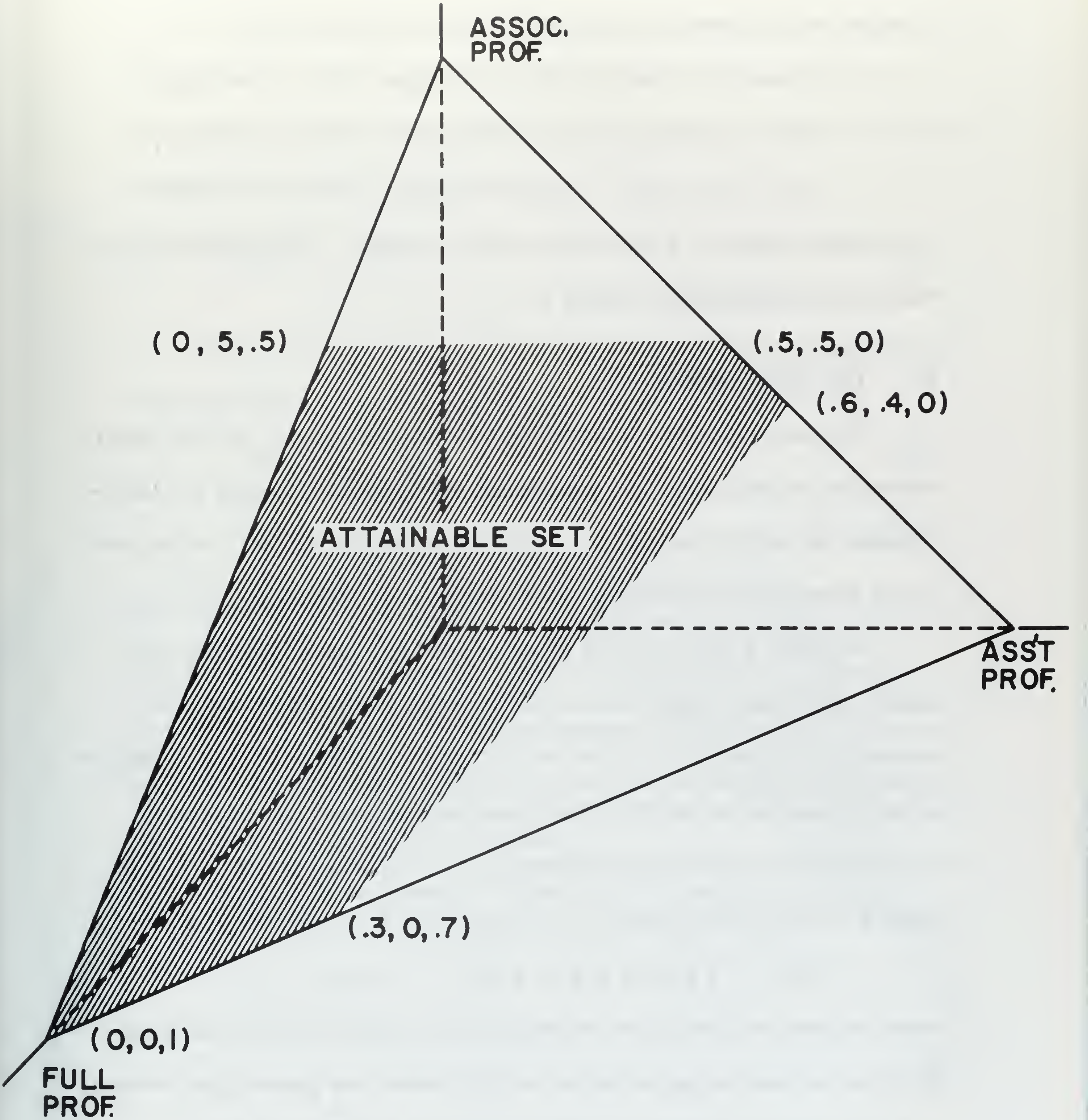

FIGURE 1.

THE ATTAINABLE SET

Equation 7 gives the following points of the Attainable set:

$$(.5 + .1, .4, 0)^*, \quad (.5, .4 + .1, 0)^*, \quad (.5, .4, 0 + .1),$$

$$(0 + .1, .6, .3), \quad (0, .6 + .1, .3)^*, \quad (0, .6, .3 + .1),$$

$$(0 + .3, 0, .7)^*, \quad (0, 0 + .3, .7), \quad (0, 0, .7 + .3)^* \ .$$

The extreme points of A are marked with an asterisk. The boundaries of this set are illustrated in Figure 1.

B. THE MAINTAINABLE SET

The set of all faculty distributions with the property that the distribution can be duplicated in consecutive time periods is called the Maintainable set and is denoted M. In the literature, this is also called the set of steady state distributions (see Bartholomew (1967)).

Let $F(t) = F$ for all t; that is we appoint the same distribution of faculty each year. From equation 3 the steady state distribution of faculty, X, is given by: $X = X \cdot P + X \cdot W \cdot F$. Let $F^*(X) = X \cdot W \cdot F$. Then F^* gives the number of faculty which must be appointed to each rank in steady state to maintain the distribution X. It follows that $F^* = X \cdot (I-P)$. Since F^* must be non-negative, we now define M as follows:

$$M = \{ X \mid X \cdot (I-P) \geq 0, X \epsilon Y \} \ .$$

Since at least one row of the P matrix sums to less than one, the matrix (I-P) has an inverse and all terms of this inverse are greater than or equal to 0 (see Kemeny and Snell, (1960)). Define $N = (I-P)^{-1}$. Then

$$X = F^* \cdot N \ . \tag{9}$$

In what follows it is more convenient to consider the fraction of new appointments to each particular rank rather than the total number. For this reason we define a new matrix, $B = (B_{ij})$, formed from the N matrix by normalizing the row sums; hence let

$$B_{ij} = \frac{N_{ij}}{\sum_k N_{ik}} \quad . \tag{9}$$

The necessary condition for X to be a member of the Maintainable set as stated in equation 8 can now be written:

$$X = F \cdot B \quad \text{, where} \tag{10}$$

$$f_i = f_i^* \cdot \sum_k N_{ik}$$

Since we require that $\Sigma X_i = 1$, it follows from the definition of B that $\Sigma f_i = 1$, the desired result. Since $f_i \geq 0$ and $\Sigma f_i = 1$, equation 10 expresses X as a convex linear combination of the rows of B. Thus the rows of B represent the extreme points of the Maintainable set which is a closed convex polyhedron in n-space.

Referring to the example of Section III A, we compute:

$$N = (I-P)^{-1} = \begin{bmatrix} 2.0 & 2.0 & 2.0 \\ 0 & 2.5 & 2.5 \\ 0 & 0 & 3.3 \end{bmatrix}$$

$$B = \frac{N_{ij}}{\sum_k N_{ik}} = \begin{bmatrix} .33 & .33 & .33 \\ 0 & .50 & .50 \\ 0 & 0 & 1.0 \end{bmatrix}$$

The Maintainable set is the indicated region of Figure 2. The extreme points of this Maintainable set are described by the three vectors defined

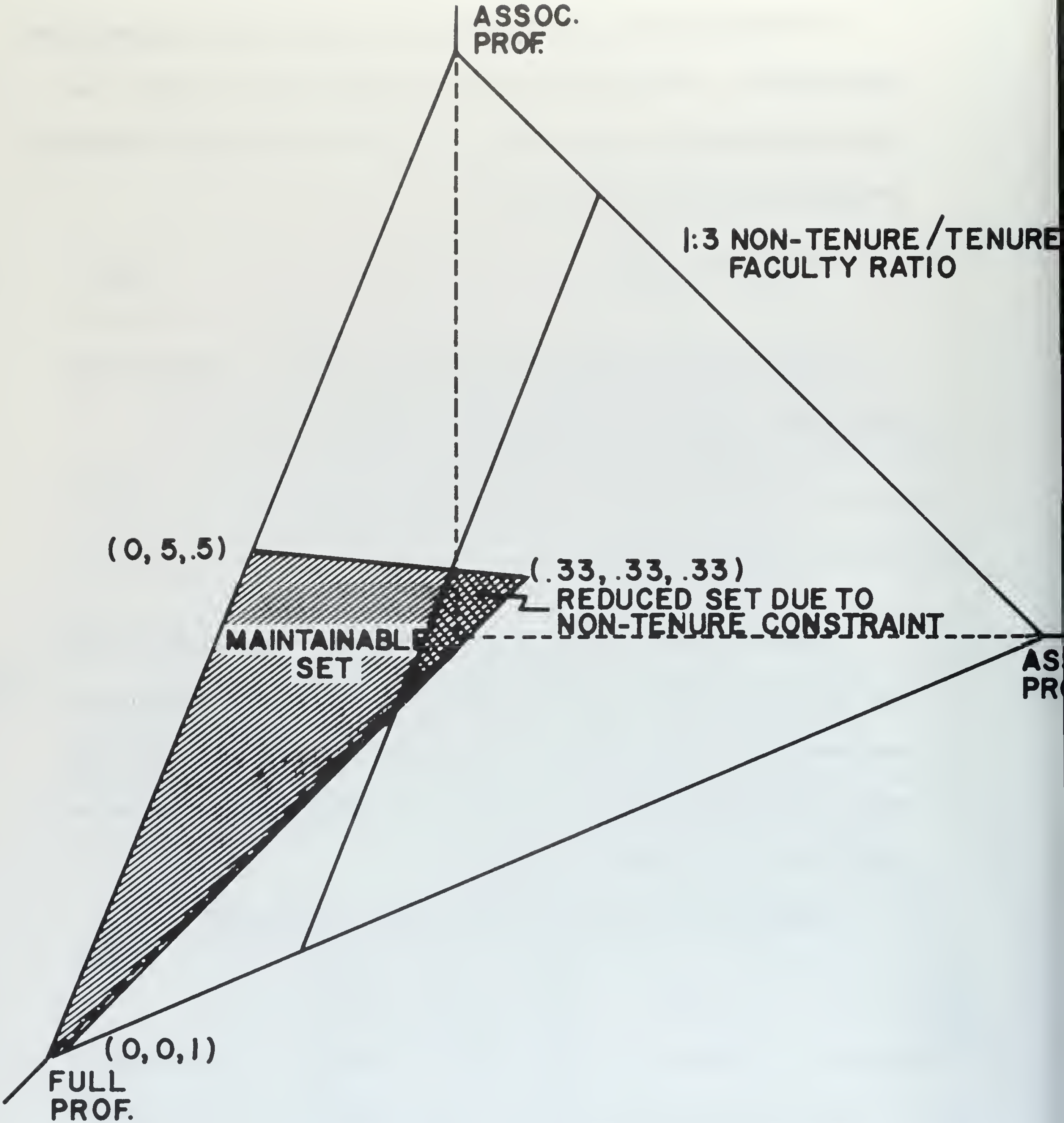

FIGURE 2.

THE MAINTAINABLE SET

by the rows of B, namely (.333, .333, .333), (0, .50, .50) and (0, 0, 1). As can be seen from Figure 2, the Maintainable set is only a small portion of the set of all distributions with a tendency toward "top heavy" faculty distributions. If a constraint is added specifying that the ratio of non-tenure faculty be no less than 1:3, the constrained Maintainable set is considerably reduced as illustrated by the cross-hatched region of Figure 2. It is easy to see that many non-tenure/tenure ratios are not maintainable.

C. THE CONTAINMENT SET

The primary reason for defining the Attainable set was to restrict the set of all distributions to those which can be reached at least once from any current distribution. We now wish to find the set of distributions which can be reached infinitely often, though not necessarily in successive time periods. We shall call this new set the Containment set and designate it by the letter C. In Section III A it was shown in equation 6 that the distributions reachable from any given starting distribution are a subset of the Attainable set. Also, the Maintainable set is a subset of the Containment set since any distribution in the Maintainable set can be achieved infinitely often with a constant appointment policy. Hence we have $M \subset C \subset A$. Mathematically we define C as follows:

$$C = \{ X \mid X \in Y,\ X \in R_n(X) \text{ for some } n \}, \text{ where}$$

$$R_n(X) = \{ X \mid X, \tilde{X} \in Y,\ X \text{ is reachable from } \tilde{X} \text{ in } n \text{ steps} \},$$

$$= \{ X \mid X, \tilde{X} \in Y,\ X = \tilde{X} \cdot P^n + \sum_i f^*(i) \cdot P^{n-i} \}$$

for some sequence of appointment vectors $\{f^*(i)\}$.

Although we have defined C, the definition provides little insight in describing the boundaries of C. Unlike the boundaries of the Maintainable and Attainable sets we are unable to provide a precise description of the boundaries of the Containment set. We conjecture that these boundaries form an infinite sided polyhedron. Some calculations in the 3-dimensional case indicate this. We proceed at this point to obtain approximations of C in the 3-dimensional case to gain some idea of its size and its relation to the Maintainable set.

It is apparent that any distribution lying outside the Maintainable set that is reachable from a point within the Maintainable set in n steps must be an element of C since return to the starting distribution is guaranteed by definition of M. Using this information, a computer program was developed to delineate those distributions reachable from points of M using appointment policies designed to keep the resulting distributions out of the Maintainable set. In the example of Section III A, the distributions resulting from appointing only Assistant Professors to an initial distribution of only Full Professors were computed. This proceedure was continued until the number of Associate Professors started to decrease in number. At that point, the appointment policy was changed to a policy of appointing only Associate Professors. It is conjectured that in the 3-dimensional case this approach gives the boundaries of C as displayed in Figure 3. It is not clear how this construction would be extended to higher dimensions.

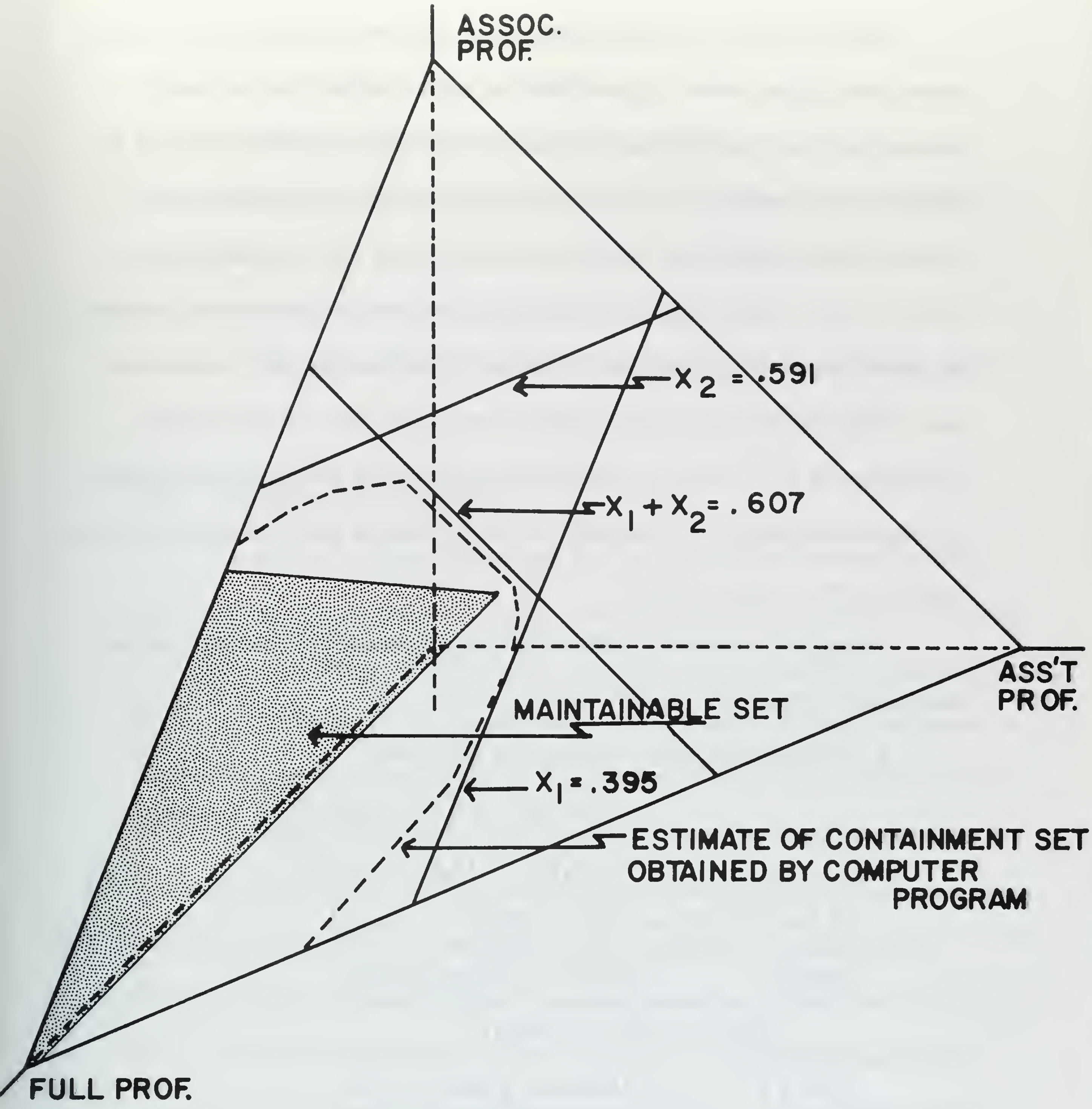

FIGURE 3

THE CONTAINMENT SET

Another approach to approximating the boundary of C in the 3-dimensional case is by means of constraint equations on the various ranks. Starting with any distribution of faculty in the Attainable set at time t we maximize the number of Assistant Professors in time t+1 subject to the constraint that the number of Assistant Professors does not decrease in years t to t+1. We can see from Figure 3 that such a proceedure provides a an upper bound on the Assistant Professor coordinate of the Containment set. A similar proceedure was used to bound the Associate Professor coordinate of C. To find a lower bound for the Full Professor co-ordinate, X_3, we maximized $X_1 + X_2$ subject to the constraint that this sum does not decrease from time t to t+1.

Using the example of Section III A, the following calculations are displayed in Figure 3.

1. Maximize $X_1(t+1) = X_1(t) \cdot P_{11} + X(t) \cdot W \cdot e_1$

$$= X_1(t) \cdot (P_{11} + P_{14}) + X_2(t) \cdot P_{24} + X_3(t) \cdot P_{34}$$

$$= .6X_1(t) + .1X_2(t) + .3X_3(t)$$

$X_1(t+1) \geq X_1(t) \Rightarrow X_1(t) \leq .25X_2(t) + .75X_3(t)$

$X(t) \in Y \qquad \Rightarrow X_3(t) = 1 - X_1(t) - X_2(t)$

$\therefore \quad X_1(t) \leq 3/7 - 2/7X_2(t)$

$X(t) \in A \qquad \Rightarrow$ Maximum $X_1(t+1) = .393$

2. Maximize $X_2(t+1) = X_1(t) \cdot P_{12} + X_2(t) \cdot P_{22} + X(t) \cdot W \cdot e_2$

$$= X_1(t) \cdot (P_{12} + P_{14}) + X_2(t) \cdot (P_{22} + P_{24}) + X_3(t) \cdot P_{34}$$

$$= .5X_1(t) + .7X_2(t) + .3X_3(t)$$

$X_2(t+1) \geq X_2(t) \Rightarrow X_2(t) \leq 5/3X_1(t) + X_3(t)$

$$X(t) \in Y \Rightarrow X_1(t) = 1 - X_2(t) - X_3(t)$$

$$\therefore \quad X_2(t) \leq 5/8 - 2/8X_3(t)$$

$$X(t) \in A \Rightarrow \text{Maximum } X_2(t+1) = .591$$

3. $$\text{Maximize } X_1(t+1) + X_2(t+1) = X_1(t) + X_2(t)\cdot(P_{22} + P_{24}) + X_3(t)\cdot P_{34}$$

$$= X_1(t) + .7X_2(t) + .3X_3(t)$$

$$X_1(t+1) + X_2(t+1) \geq X_1(t) + X_2(t) \Rightarrow X_3(t) \geq X_2(t)$$

$$X(t) \in A \text{ and Maximum } X_1(t+1) = .393 \Rightarrow \text{Maximum } X_1(t+1) + X_2(t+1) = .607$$

D. APPOINTMENT POLICIES

Up to now we have discussed the set of distributions attainable under various appointment policies. In this section we discuss the types of appointment policies necessary to achieve certain faculty distributions. Since distributions in the Maintainable set can be maintained by means of a constant appointment policy as shown in equation 10, it will be assumed that the desired distribution lies outside the Maintainable set but within the Containment set. Since distributions in this region are not maintainable from time period to time period, the sequence of distributions realized during a repeating cycle contains elements of both the Containment set and the Maintainable set. We limit our study of appointment policies to considering their effect on distributions in the Maintainable set since these effects are easier to analyze. Let X be the steady state distribution of faculty such that X_i = fraction of total faculty in rank i. Recall from equation 10 that $X = F \cdot B$.

Due to the upper triangular feature of the promotion matrix and the resulting similar configuration of the N and B matrices, the effect of making all new appointments at one level (i.e. making F a unit vector) will realize a distribution of faculty as described by the row of the B matrix corresponding to the level of appointments. This means that a certain faculty rank cannot be maintained at a zero level, or the probability of promotion is zero. Mathematically, this becomes:

$$X_j \geq \frac{1}{1-P_{jj}} \left(\sum_i X_i \cdot P_{ij} \right) \quad \text{, and so}$$

$$\begin{aligned} X_j = 0 \Rightarrow \text{ either } X_i &= 0 \qquad i < j \\ \text{or } P_{ij} &= 0 \qquad i < j \,. \end{aligned}$$

In order to simplify analysis of the model we rule out the possibility of double and triple promotions. With this simplification, the above inequality reduces to:

$$X_j \geq \frac{1}{1-P_{ij}} \cdot X_{j-1} \cdot P_{j-1,j} \quad .$$

If the probability of remaining in a particular rank plus the probability of being promoted into that rank sums to more than one, then the number of faculty in that rank must be greater than the number in the preceding rank. Mathematically, if

$$P_{j-1,j} + P_{j,j} > 1 \Rightarrow X_j > X_{j-1}, \quad \text{and if}$$

$$P_{j-1,j} + P_{j,j} = 1 \Rightarrow X_j \geq X_{j-1} \quad .$$

Since a rank can only be maintained by appointing at that rank or at lower ranks, it is necessary to make some portion of the total appointments at the lowest rank in order that all ranks be represented. This point is obvious if one stops to consider the implications of the promotion matrix.

Consider the sixteen states defined on page 4. For the largest ratio of non-tenure faculty to total faculty (where non-tenure faculty includes Assistant Professor, steps 1-4), appointments should be made exclusively at that rank where the sum of the first four elements of the B matrix is maximum. The above appointment policy statement can thus be expressed as follows: appoint to state i, where i is chosen so that

$$\sum_{j=1}^{4} B_{ij} \geq \sum_{j=1}^{4} B_{kj} \quad \text{for all } k = 1,2,\ldots,n \ .$$

For the case where only single promotion advancements are permitted, appointments should be made exclusively at the Assistant Professor Step 1 level since:

$$\sum_{j=1}^{4} B_{ij} \geq \sum_{j=1}^{4} B_{kj} \quad i < k \ .$$

Proof:

$$\sum_{j=1}^{13} B_{ij} = 1 \quad \text{since B is normalized.}$$

$$\therefore \quad \sum_{j=1}^{4} B_{ij} = 1 - \sum_{j=5}^{13} B_{ij} \quad \text{for all } i = 1,2,\ldots,n \ .$$

For single promotion only, it can be shown that

$$B_{ij} = \frac{P_{i,i+1}}{1-P_{i,i}} \cdot B_{i+1,} \quad .$$

If $P_{ii} + P_{i,i+1} = 1 \Rightarrow B_{ij} = B_{i+1,j}$, and if

$P_{ii} + P_{i,i+1} < 1 \Rightarrow B_{ij} < B_{i+1,j}$.

Therefore, $\sum_{j=5}^{13} B_{ij} \leq \sum_{j=5}^{13} B_{i+1,j}$ and hence

$$\sum_{j=1}^{4} B_{ij} \geq \sum_{j=1}^{4} B_{kj} \qquad i < k \ .$$

Thus, one can increase the non-tenure/tenure ratio by appointing at the Assistant Professor Step 1 level and reduce this ratio by appointing at the Assistant Professor Step 4 level. Such effects of appointment policy are, however, predicated on the fact that the transition matrix will remain essentially unchanged. The effect of changes in the transition matrix will be discussed in the next chapter.

The effect of a mixed appointment policy is easily determined once the B matrix is known. A policy of appointing 20% of the new appointments to tenure positions could be evaluated by taking 80% of row 1 of the B matrix (corresponding to appointment at the Assistant Professor Step 1 level) and 20% of row 5 of the B matrix (corresponding to appointments at the Associate Professor Step 1 level). Such an appointment schedule will realize the greatest achievable spread among the various ranks, given the constrained appointment policy. Other appointment policies can be evaluated in a similar way.

IV. EFFECT OF PROMOTION POLICIES

Although the preferred method of regulating faculty distributions is by means of an appointment policy, it is evident from the preceding chapter that other means must be employed if certain distributions are to be realized. One such method involves changing the promotion and/or retirement policies of the institution. For example, if a general policy of requiring faculty members to remain a minimum of two years in each grade were altered to a three year minimum, the change would have a noticeable effect on the transition matrix and subsequently on the boundaries of the Maintainable, Attainable and Containment sets. Although the exact effect that policy changes have on transition frequencies is not known, these relationships can be determined with reasonable accuracy by analysing past transition matrices both before and after such changes are implemented.

In this section, we examine how changes to the promotion matrix affect maintainable faculty distributions by first studying individual transition frequency changes and then looking at changes to all the transition frequencies associated with a given state. In order to simplify the analysis and to focus attention on major effects, we consider only single promotions as a means of advancement. Although jump promotions are possible, their relative occurrence is small and do not appreciably alter the results of the analysis.

With the above restriction, the elements of the N matrix ($= (I-P)^{-1}$) can be expressed as follows:

$$N_{ij} = \frac{1}{P^*_{jj}} \prod_{k=i}^{j-1} \frac{P^*_{k,k+1}}{P^*_{k,k}} \qquad \text{for } j \geq i \,,$$

$$= 0 \qquad \text{for } j < i \,,$$

where $P^*_{jj} = 1 - P_{jj}$, and

$P^*_{ij} = P_{ij}$.

This can be rewritten in recursive form:

$$N_{ij} = N_{i,j-1} \cdot \frac{P^*_{j-1,j}}{P^*_{j,j}} \qquad \text{for } j \geq i \,,$$

$$= \frac{1}{P^*_{jj}} \qquad \text{for } j = i, \text{ and}$$

$$= 0 \qquad \text{otherwise.}$$

In this form it is more apparent what effect a change in promotion policy has on the distribution of faculty.

If the probability of remaining in state j, given that one starts in state j, is increased (P_{jj} new > P_{jj} old), then the proportion of faculty in states j through n will increase and the ratio of faculty in states 1 through j-1 will decrease, given that some appointments are made at the j-1 level or below. This is because an increase in P_{jj} results in a decrease of $1-P_{jj}$ or an increase in $\frac{1}{1-P_{jj}}$. Therefore, in the N matrix, all states from j through n will be increased proportional to $\frac{1}{1-P_{jj}}$, while states 1 through j-1 will remain unchanged; this results in changed ratios of faculty in the different ranks. The opposite result holds if P_{jj} is decreased.

If the probability of being promoted to state j+1, given one starts in state j, is increased, the ratio of faculty in states j+1 through n will increase, and the ratio of faculty in states 1 through j will decrease, given that some appointments are made at the j level or below. This is because an increase in P_{ij} results in an increase in $\frac{P_{ij}}{1-P_{jj}}$. Thus in the N matrix, all states from j+1 through n will be increased by an amount $\frac{P_{ij}}{1-P_{jj}}$, while states 1 through j will remain unchanged. The opposite result again holds for a decrease in P_{ij}.

When both the probability of promotion $P_{j,j+1}$, and the probability of remaining in state j, $P_{j,j}$, are increased, the overall effect is to increase the ratio of faculty in states j+1 through n, given that appointments are made at the j level or below. The effect on the ratio of faculty in state j is dependent upon the relative increases in probabilities and upon the appointment policy. If appointments are limited to state j and above, the ratio of faculty in state j will decrease. If the increase in P_{jj} is much greater than the increase in $P_{j,j+1}$, and appointments are made at the state j-1 level or below, then some increase in the ratio of faculty for state j may be realized, but at no time will the increase be as large for state j as for higher states.

When the probability of remaining in state j is increased while the probability of being promoted from state j is decreased, the overall effect on faculty distribution is to increase the proportion of faculty in state j and decrease the proportion of faculty in states j+1 through n. The effect

on states 1 through j-1 is dependent upon the size of the increase or decrease relative to the initial probabilities and the subsequent appointment policy.

V. A CASE STUDY

The following sections describe the data, transition matrix, and a comparison of predicted and actual distributions resulting from a case study of the College of Engineering at the University of California at Berkeley.

A. COLLECTION OF DATA

The data on faculty movements within the College of Engineering was gathered by examining College and University records for discrete time points in the past, namely at 12:01 A.M. on 1 July of the years 1960 through 1968. Faculty in a particular state at these points in time were considered to remain in that state throughout the one year time period. The data was collected in the following manner:

1. Historical records of all presently employed faculty members of the College of Engineering were obtained by the Dean's Office. The present age and time spent in each position was recorded for each faculty member, and accumulated for each department in the college. This information was then used to form a historical record of the faculty distribution of the various departments, and for the College of Engineering as a whole, from 1960 through 1968. At no time were confidential personnel records or biographical data made available to us.
2. In conjunction with the above, data was also collected at University Hall to check the lists of faculty of the College of Engineering for the years 1960 through 1968. Starting in 1960,

the listing of faculty members by department was recorded, and this list was updated through 1968. The record of faculty members currently holding positions in the College of Engineering was compared with the listing from the Dean's Office to provide an independent check of promotion data. In addition, a listing was made of faculty members who were employed sometime during the period 1960 to 1967 but who are not presently employed, in order to provide a historical record of the entire College during this period.

3. A separate listing was provided by the Dean's Office recording the total numbers and distribution of faculty from 1960 through 1968, as ascertained from college records. Included with this list was a breakdown by name of those faculty members no longer employed by the University, and why and when they terminated employment. This information was used as a check against the other two sources of information. Only in a few special cases was the data from these sources in conflict; these discrepancies were resolved by corresponding directly with the Dean's Office.

4. A final source of information of the College of Engineering was provided by an independent study conducted by the Chancelor's Office. Using information gathered from University files as input data, a computer program designed by personnel of the Chancellor's Office was used to provide a complete profile of

the College of Engineering by age and position for each year from 1960 through 1968. Although this information was gathered from different records than those used in this report, and a different time base was used, the overall results of this report agree quite closely with this profile listing.

A compilation of the raw data is presented in Tables 1 through 8. The cumulative data for the period 1960 through 1968 is presented in Table 9.

B. THE TRANSITION MATRIX

The transition matrix for the College of Engineering was calculated in the following manner:

1. The total number of faculty in each position for a particular year was determined by summing the statistics of the individual departments within the College.
2. In a similar manner, the total number of faculty in each position that were given merit increases, promoted or accelerated as well as those who resigned, retired, or died during a particular year, were recorded.
3. In order to provide a larger sample base for estimating parameters, the annual statistics for the period 1960 through 1968 were aggregated to form totals representing a single period. This procedure is justified if the transition process is in fact Markovian, and if no major policy changes occurred during

this time period that could effect the transition process. This author was able to discover only one policy change during the period that might have affected the transition process: the addition of new steps to the Assistant Professor and Full Professor ranks in the period 1962-1963. A straightforward aggregation of the data from 1960-1968 would indicate an unrealistic double promotion rate from Assistant Professor, Step 3 to Associate Professor, Step 1, since the Assistant Professor, Step 4 state did not exist from 1960-1963. We are not concerned in this paper with the actual policies used by the College of Engineering during the transition period. Rather, we must devise some method to estimate current realistic transition frequencies, P_{33}, P_{34}, P_{44}, and P_{45} from past data. We accomplished this by considering all Assistant Professors, Step 3 to be Assistant Professors, Step 4 in the period 1960-1963. Thus, all promotions from Assistant Professor, Step 3 to Associate Professors, Step 1 in this period were single promotions. Also, in the period 1960-1963, since Full Professor, Step 5, did not exist, straightforward aggregation of data would lead to unrealistic estimates of current promotion rates from Full Professor, Step 4, to the overscale level. By assuming all Full Professors, Step 5, were overscale in 1960-1963 we were able to obtain more realistic transition frequencies from Full Professor, Step 4 to Full Professor, Step 5.

4. The elements of our transition matrices were derived by determining the observed relative frequency of transitions between states. Thus, the fraction of Assistant Professors, Step 1, that become Assistant Professor, Step 2, in the next year is equal to the observed number of promotions from state 1 to state 2 divided by the total number in state 1 at the beginning of the year.

The transition matrix based upon the data from the years 1960 through 1965 is shown in Figure 4.

C. COMPARISON OF PREDICTED AND ACTUAL DISTRIBUTIONS

In order to determine how well the model predicts actual faculty distributions, data from the years 1960 through 1965 were used to determine transition estimates for the promotion matrix. Since these transition estimates agree quite closely with the estimates based upon the nine year period 1960 through 1968, the Markov assumption appears to be reasonably well justified. The distribution of faculty in 1 July 1965 was used to calculate returning faculty. New appointments were added to predict the distribution of faculty as of 1 July 1966. This procedure was repeated for subsequent years, using the previous years predicted distribution plus the actual appointment policy of the current year to determine the current distribution of faculty. The transition matrix based upon the data from the years 1960 through 1965 is shown in Figure 5. The predicted distribution of faculty for the years 1966 - 1968, based

upon the 1965 distribution of faculty and the ensuing appointment schedules, is shown in Figure 6 along with the actual distributions for these years. In a three-year average, the estimate of Assistant Professors was 4.25% below the actual, the estimate of Associate Professors was 10% above the actual, the estimate of Full Professors was 4.9% below the actual. The estimated total number of faculty for each year was, on the average, 1.7% below the actual total number of faculty.

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1		6															6
(2)	Ass't Prof.	Step 2		3	2														5
(3)	Ass't Prof.	Step 3			4		2										1		7
(4)	Ass't Prof.	Step 4				0													0
(5)	Assoc. Prof.	Step 1					8	6											14
(6)	Assoc. Prof.	Step 2						9	2										11
(7)	Assoc. Prof.	Step 3							27	9									36
(8)	Full Prof.	Step 1								11	4					1			16
(9)	Full Prof.	Step 2									14	2							16
(10)	Full Prof.	Step 3										28	4						32
(11)	Full Prof.	Step 4											4						4
(12)	Full Prof.	Step 5												8					8
(13)	Full Prof.	O/S													0				0
	Appointments		1	1	2	0	0	3	2	0	0	0	0	0	0				9
	Total (1961)		1	10	8	0	10	18	31	20	18	30	8	8	0	1	1	0	164

TABLE 1: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 July 1960 to 1 July 1961

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1	1																1
(2)	Ass't Prof.	Step 2		4	5	1													10
(3)	Ass't Prof.	Step 3			2		6												8
(4)	Ass't Prof.	Step 4				0													0
(5)	Assoc. Prof.	Step 1					4	6											10
(6)	Assoc. Prof.	Step 2						10	8										18
(7)	Assoc. Prof.	Step 3							17	13							1		31
(8)	Full Prof.	Step 1								15	5								20
(9)	Full Prof.	Step 2									14	4							18
(10)	Full Prof.	Step 3										19	9			1	1		30
(11)	Full Prof.	Step 4											8						8
(12)	Full Prof.	Step 5												8					8
(13)	Full Prof.	O/S													0				0
	Appointments		1	5	1	0	0	0	1	1	0	0	0	0	0				9
	Total (1962)		2	9	8	1	10	16	26	29	19	23	17	8	0	1	2	0	171

TABLE 2: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1961 TO 1 July 1962

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1		2															2
(2)	Ass't Prof.	Step 2		6	3														9
(3)	Ass't Prof.	Step 3			2	0	6												8
(4)	Ass't Prof.	Step 4				1													1
(5)	Assoc. Prof.	Step 1					7	3											10
(6)	Assoc. Prof.	Step 2						7	9										16
(7)	Assoc. Prof.	Step 3							16	8							2		26
(8)	Full Prof.	Step 1								21	8								29
(9)	Full Prof.	Step 2									13	6							19
(10)	Full Prof.	Step 3										17	5				1		23
(11)	Full Prof.	Step 4											13	4					17
(12)	Full Prof.	Step 5												5	2		1		8
(13)	Full Prof.	O/S													0				0
	Appointments		5	4	4	2	1	0	0	0	0	0	0	0	0				16
	Total (1963)		5	12	9	3	14	10	25	29	21	23	18	9	2	0	4	0	184

TABLE 3: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1962 TO 1 JULY 1963

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1	4	0	1														5
(2)	Ass't Prof.	Step 2		4	5	2											1		12
(3)	Ass't Prof.	Step 3			7	1	1												9
(4)	Ass't Prof.	Step 4				1	1										1		3
(5)	Assoc. Prof.	Step 1					7	5	2										14
(6)	Assoc. Prof.	Step 2						6	4										10
(7)	Assoc. Prof.	Step 3							17	8									25
(8)	Full Prof.	Step 1								19	8	2							29
(9)	Full Prof.	Step 2									18	3							21
(10)	Full Prof.	Step 3										18	3			1	1		23
(11)	Full Prof.	Step 4											18						18
(12)	Full Prof.	Step 5												8	1				9
(13)	Full Prof.	O/S													2				2
	Appointments		1	3	3	0	0	0	2	1	0	0	0	0	0				10
	Total (1964)		5	7	16	4	9	11	25	28	26	23	21	8	3	1	3	0	190

TABEL 4: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1963 TO 1 JULY 1964

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1		3													2		5
(2)	Ass't Prof.	Step 2		4	1	1											1		7
(3)	Ass't Prof.	Step 3			6	3	4										2	1	16
(4)	Ass't Prof.	Step 4				2	2												4
(5)	Assoc. Prof.	Step 1					3	3	3										9
(6)	Assoc. Prof.	Step 2						6	4								1		11
(7)	Assoc. Prof.	Step 3							19	6									25
(8)	Full Prof.	Step 1								16	8	1	1				2		28
(9)	Full Prof.	Step 2									13	9				1	2	1	26
(10)	Full Prof.	Step 3										16	6				1		23
(11)	Full Prof.	Step 4											11	9	1				21
(12)	Full Prof.	Step 5												7	1				8
(13)	Full Prof.	O/S													2		1		3
	Appointments		3	8	3	1	1	0	0	0	0	0	0	0	0				16
	Total (1965)		3	15	10	7	10	9	26	22	21	26	18	16	4	1	12	2	202

TABLE 5: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1964 TO 1 JULY 1965

RANK AND STEP

			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	Ret. (14)	Res. (15)	Dec. (16)	Total
(1)	Ass't Prof.	Step 1	2	1															3
(2)	Ass't Prof.	Step 2		5	9												1		15
(3)	Ass't Prof.	Step 3			4	2	3										1		10
(4)	Ass't Prof.	Step 4				3	2										2		7
(5)	Assoc. Prof.	Step 1					7	2	1										10
(6)	Assoc. Prof.	Step 2						6	2	1									9
(7)	Assoc. Prof.	Step 3							14	11							1		26
(8)	Full Prof.	Step 1								17	5								22
(9)	Full Prof.	Step 2									16	4				1			21
(10)	Full Prof.	Step 3										16	9				1		26
(11)	Full Prof.	Step 4											14	3			1		18
(12)	Full Prof.	Step 5												10	5		1		16
(13)	Full Prof.	O/S													4				4
	Appointments		1	5	6	0	0	1	2	0	1	0	0	0	0				16
	Total (1966)		3	11	19	5	12	9	19	29	22	20	23	13	9	1	8	0	203

TABLE 6: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1965 TO 1 JULY 1966

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1		2	0	1													3
(2)	Ass't Prof.	Step 2		6	4	1													11
(3)	Ass't Prof.	Step 3			11	7	1												19
(4)	Ass't Prof.	Step 4				3	2												5
(5)	Assoc. Prof.	Step 1					5	7											12
(6)	Assoc. Prof.	Step 2						5	4										9
(7)	Assoc. Prof.	Step 3							10	9									19
(8)	Full Prof.	Step 1								20	9								29
(9)	Full Prof.	Step 2									15	7							22
(10)	Full Prof.	Step 3										15	4				1		20
(11)	Full Prof.	Step 4											17	6					23
(12)	Full Prof.	Step 5												8	5				13
(13)	Full Prof.	O/S													9				9
	Appointments		0	5	2	1	2	1	3	0	0	0	0	0	0				14
	Total (1967)		0	13	17	13	10	13	17	29	24	22	21	14	14	0	1	0	208

TABLE 7: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1966 TO 1 JULY 1967

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1	0																0
(2)	Ass't Prof.	Step 2		2	10												1		13
(3)	Ass't Prof.	Step 3			3	13	1												17
(4)	Ass't Prof.	Step 4				3	8	2											13
(5)	Assoc. Prof.	Step 1					1	6	3										10
(6)	Assoc. Prof.	Step 2						6	7										13
(7)	Assoc. Prof.	Step 3							10	6	1								17
(8)	Full Prof.	Step 1								16	13								29
(9)	Full Prof.	Step 2									12	7	4			1			24
(10)	Full Prof.	Step 3										12	8	1				1	22
(11)	Full Prof.	Step 4											10	10		1			21
(12)	Full Prof.	Step 5												11	3				14
(13)	Full Prof.	O/S													13		1		14
	Appointments		1	2	4	0	1	0	0	0	0	0	0	0	0	.			8
	Total (1968)		1	4	17	16	11	14	20	22	26	19	22	22	16	2	2	1	215

TABLE 8: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1967 TO 1 JULY 1968

RANK AND STEP			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14) Ret.	(15) Res.	(16) Dec.	Total
(1)	Ass't Prof.	Step 1	7	14	1	1											2		25
(2)	Ass't Prof.	Step 2		34	39	5											4		82
(3)	Ass't Prof.	Step 3			39	26	24										4	1	94
(4)	Ass't Prof.	Step 4				13	15	2									3		33
(5)	Assoc. Prof.	Step 1					42	38	9										89
(6)	Assoc. Prof.	Step 2						55	40	1							1		97
(7)	Assoc. Prof.	Step 3							130	70	1						4		205
(8)	Full Prof.	Step 1								135	60	3	1			1	2		202
(9)	Full Prof.	Step 2									120	44	4			3	2	1	174
(10)	Full Prof.	Step 3										141	48	1		2	6	1	199
(11)	Full Prof.	Step 4											95	32	1		2		130
(12)	Full Prof.	Step 5												65	17		2		84
(13)	Full Prof.	O/S													30		2		32
	Appointments		13	33	25	4	6	6	6	4	1	0	0	0	0				98
	Total (1968)		20	81	104	49	87	101	185	210	182	188	148	98	48	6	34	3	1544

TABLE 9: MOVEMENT OF FACULTY FOR THE COLLEGE OF ENGINEERING AT BERKELEY FOR THE PERIOD 1 JULY 1960 TO 1 JULY 1968 ACCUMULATED STATISTICS

STATE I/J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.28	.56	.04	.04	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.08	.00
2	.00	.41	.47	.07	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.05	.00
3	.00	.00	.44	.37	.14	.00	.00	.00	.00	.00	.00	.00	.00	.00	.04	.01
4	.00	.00	.00	.38	.52	.04	.00	.00	.00	.00	.00	.00	.00	.00	.07	.00
5	.00	.00	.00	.00	.47	.43	.10	.00	.00	.00	.00	.00	.00	.00	.00	.00
6	.00	.00	.00	.00	.00	.57	.41	.01	.00	.00	.00	.00	.00	.00	.01	.00
7	.00	.00	.00	.00	.00	.00	.64	.34	.00	.00	.00	.00	.00	.00	.02	.00
8	.00	.00	.00	.00	.00	.00	.00	.67	.30	.01	.00	.00	.00	.00	.01	.00
9	.00	.00	.00	.00	.00	.00	.00	.00	.69	.25	.02	.00	.00	.02	.01	.01
10	.00	.00	.00	.00	.00	.00	.00	.00	.00	.71	.24	.01	.00	.01	.03	.01
11	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.73	.25	.01	.01	.01	.00
12	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.77	.20	.00	.02	.00
13	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.97	.00	.03	.00
14	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00	.00	.00
15	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00	.00
16	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00

Figure 4: TRANSITION MATRIX FROM 1960 - 1968 DATA

STATE I/J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.32	.54	.05	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.09	.00
2	.00	.45	.43	.07	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.05	.00
3	.00	.00	.44	.37	.14	.00	.00	.00	.00	.00	.00	.00	.00	.00	.04	.01
4	.00	.00	.00	.38	.47	.06	.00	.00	.00	.00	.00	.00	.00	.00	.09	.00
5	.00	.00	.00	.00	.54	.37	.05	.00	.00	.00	.00	.00	.00	.00	.04	.00
6	.00	.00	.00	.00	.00	.59	.39	.01	.00	.00	.00	.00	.00	.00	.01	.00
7	.00	.00	.00	.00	.00	.00	.65	.33	.00	.00	.00	.00	.00	.00	.02	.00
8	.00	.00	.00	.00	.00	.00	.00	.69	.26	.02	.01	.00	.00	.01	.01	.00
9	.00	.00	.00	.00	.00	.00	.00	.00	.73	.23	.00	.00	.00	.02	.02	.01
10	.00	.00	.00	.00	.00	.00	.00	.00	.00	.73	.23	.00	.00	.01	.03	.00
11	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.78	.19	.02	.00	.01	.00
12	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.81	.16	.00	.03	.00
13	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.89	.00	.11	.00
14	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00	.00	.00
15	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00	.00
16	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	1.00

Figure 5: TRANSITION MATRIX FROM 1960 - 1965 DATA

		1965	1966			1967			1968		
		Actual	Appointed	Predicted	Actual	Appointed	Predicted	Actual	Appointed	Predicted	Actual
Ass't. Prof.	Step 1	3	1	2	3	--	1	--	1	1	1
	Step 2	15	5	13	11	5	12	13	2	8	4
	Step 3	10	6	17	19	2	15	17	4	16	17
	Step 4	7	--	7	5	1	11	13	0	11	16
	Total	35	12	39	38	8	39	43	7	36	38
Assoc. Prof.	Step 1	10	--	10	12	2	13	10	1	15	11
	Step 2	9	1	10	9	1	11	13	--	12	14
	Step 3	26	2	23	19	3	23	17	--	20	20
	Total	45	3	43	40	6	47	40	1	47	45
Full Prof.	Step 1	22	--	24	29	--	24	29	--	24	22
	Step 2	21	1	22	22	--	22	24	--	23	26
	Step 3	26	--	24	20	--	23	22	--	23	19
	Step 4	18	--	20	23	--	22	21	--	22	22
	Step 5	16	--	16	13	--	17	14	--	18	22
	Overscale	4	--	7	9	--	9	14	--	11	16
	Total	107	1	113	116	--	117	124	--	121	127
	Total	187	16	195	194	14	203	207	8	204	210

Figure 6: PREDICTED AND ACTUAL FACULTY DISTRIBUTION FOR YEAR 1966-1968

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13. ABSTRACT

A Markov Model is developed to study the distribution of faculty in a university. The purpose of this paper is to determine the underlying factors that control the distribution of faculty members from one time period to the next. Maintainable and realizeable distributions are defined. The effects of appointment policies and promotion and retirement policies in controlling distributions are discussed. The model is tested by comparing the predicted output with real data.

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